The impact of hypocrisy on opinion formation: A dynamic model

Zhao Chi(Not author, just presenter)

St.Petersburg State University
Faculty of Applied Mathematics and Control Processes
dandanv5@hotmail.com

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Introduction

Opinion Dynamics focus on the generation, diffusion and aggregation of opinions or behaviors in a social network. Humans have a demonstrated tendency to copy or imitate the behavior and attitude of others and actively influence each other's opinions. But, publicly revealed opinions are not necessarily in line with internal opinions, causing complex social influence dynamics.

Basic Voter Model(BVM)[4]

BVM represents the opinions of individuals as a binary variable in a single opinion dimension. At each update, an individual adopts the opinion of one of his neighbors.

However, the fact is due to various reasons. Many people are often reluctant to express their true opinion in public. Therefore, opinions are divided into two types, **internal opinions** and **external opinions**.

Concealed Voter Model(CVM)[3, 2]

People express their external opinion publicly and keep their internal opinion.

Introduction

Hypocrisy

We call those who have different internal and external opinion **Hypocrisy**.

All instances of hypocrisy create **cognitive dissonance**[1], which can be reduced mainly in two ways: **internalization** and **externalization**.

internalization

The individual accepts the opinion that he has expressed publicly.

externalization

The previously internal opinion becomes publicly expressed.

Introduction

The BVM and CVM share the following assumptions:

Assumptions

- There are only two kinds of potential opinions;
- In the external layer, individuals interact in pairs.
- The group is **homogeneous**, i.e. each rate is the same for all individuals at all times;
- Two kinds of opinions have the same transition rates.

Introduction

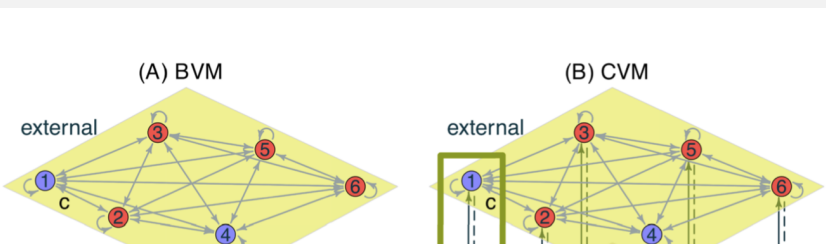

Figure 1: BVM vs CVM

Where c, e, i represent the copy, externalization and internalization rate respectively. And both in BVM and CVM, we model the external layer as a complete graph.

Notations

- R: external red;
- r: internal red;
- B: external blue;
- b: internal blue;
- ρ_x: the fraction of agents with opinion x, $x \in \{B, R\}$;
- ρ_{Xy}: the fraction of agents with external opinion X and internal opinion y, where $X \in \{R, B\}$ and $y \in \{r, b\}$.

For BVM, if $\rho_B = 1$ or $\rho_R = 1$, we say the group **reaches a consensus**.
For CVM, the group **reaches a consensus** when $\rho_{Rr} = 1$ or $\rho_{Bb} = 1$

Monte Carlo Simulation of BVM with N individuals

1. **Initialization**: Given fraction ρ_R of the nodes are red. All other nodes are blue. And initialize the time $t \to 0$.
2. **Iteration**
   a. Choose a "focal" individual f uniformly at random from all of the N individuals.
   b. Pick a neighbor n of the focal individual uniformly at random from all of its neighbors.
   c. f adopts $n's$ opinion.
   d. We increase the time by a random number Δt drawn from an exponential distribution with mean $\frac{1}{cN}$, $t \to t + \Delta t$.
   e. If the group have reached a consensus, we set $t \to T_{\text{cons}}^{(\text{BVM})}$ and terminate. Otherwise we go back to step 1.

Monte Carlo Simulation of CVM with N individuals

1. **Initialization**:
   - a given fraction ρ_{Rb} of the individuals is externally red and internally blue;
   - a given fraction ρ_{Br} is externally blue and internally red
   - a given fraction ρ_{Rr} is red in both layers
   - the rest is blus in both layers

   And initialize the time $t \to 0$.
2. **Iteration**
   a. Choose a "focal" individual f uniformly at random from all of the N individuals.
   b. Pick a neighbor n of the focal individual uniformly at random from all of its neighbors. And generate a random number u that is uniformly distributed between 0 and $c + e + i$. We have three cases there.
      - **Copying**, if $u < c$;
      - **Externalization**, if $c \leq u < c + e$;
      - **Internalization**, otherwise.

   c. We increase the time by a random number Δt drawn from an exponential distribution with mean $\frac{1}{(c+e+i)N}$, $t \to t + \Delta t$.
   d. If the group have reached a consensus, we set $t \to T_{\text{cons}}^{(\text{CVM})}$ and terminate. Otherwise we go back to step 1.

Simulation in a small group $N = 16$

The following figure shows the simulation with parameters $N = 16, c = 1, e = 1/4, i = 1/16$

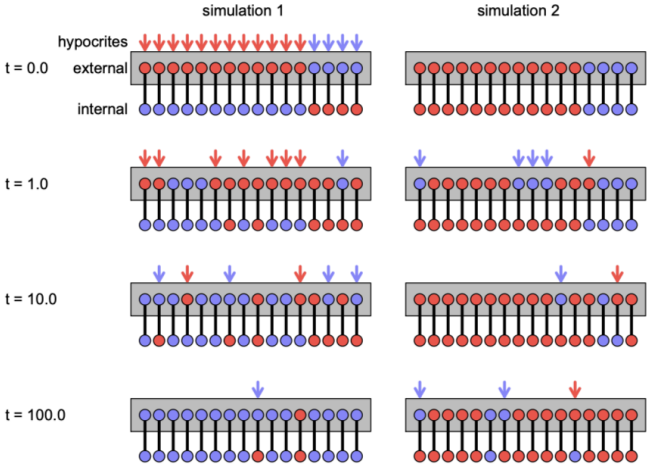

Figure 2: Two illustrative simulation

Consensus time in the BVM and CVM

Both the BVM and the CVM lead to a consensus within a finite time, provided that the number of individuals N is finite. The only exception occurs when $i = 0$ in the CVM and both r and b are present in the internal layer. In that special case, r and b persist for an infinitely long time.[2]

The mean consensus time of BVM in the complete graph[2]

$$T_{\text{cons}}^{(\text{BVM})}(\rho_R) = -\frac{N}{c}\left[\rho_R \ln \rho_R + (1-\rho_R)\ln(1-\rho_R)\right] \tag{1}$$

Define the function:

$$m(\rho_R, \rho_r) = \frac{i\rho_R + e\rho_r}{e+i} \tag{2}$$

We can view m as the **overall strength** of the red opinion that is present in the combination of the external and internal layer. m is a **martingale**, this proof can be found in [2].

Consensus time in the BVM and CVM

martingale

A continuous-time martingale with respect to the stochastic process X_t is a stochastic process Y_t such that for all t:

$$\begin{aligned} \mathbf{E}(|Y_t|) &< \infty \\ \mathbf{E}(Y_t \mid \{X_\tau, \tau \leq s\}) &= Y_s \quad \forall s \leq t \end{aligned} \tag{3}$$

The mean consensus time $T_{\text{cons}}^{(\text{cvm})}$ in the CVM can be directly obtained from that in the BVM,

$$T_{\text{cons}}^{(\text{CVM})}(m) = \tau(c, e, i) \cdot T_{\text{cons}}^{(\text{BVM})}(m) \tag{4}$$

where

$$\tau(c, e, i) = \frac{(c + e + i)(e + i)^2}{i\left[(e + i)^2 + ci\right]} \tag{5}$$

Consensus time in the BVM and CVM

Because c, e and i are positive numbers. We have:

$$
\begin{aligned}
& 0 < c < c + e \quad \text{and} \quad 0 < i^2 < (e + i)^2 \\
& \Rightarrow 0 < ci^2 < (c + e)(e + i)^2 \\
& \Rightarrow 0 < i\left[(e + i)^2 + ci\right] < (c + e + i)(e + i)^2
\end{aligned}
$$

Therefore, $\tau(c, e, i) > 1$, which means consensus time is always longer in the CVM than in the BVM. We can see from equations (5, 4 and 2), that the mean consensus time T_{cons}^{CVM} are fully determined by c, e, i, n, ρ_R and ρ_r. And do not depend on the amount of hypocrites ρ_{Rb} or ρ_{Br}.

Consensus formation in the CVM

Authors did some numerical simulation with parameters $N = 400, c = 1, e = 1/4$ and $i = 1/16$. These simulations shown that the process of consensus formation takes place in two stages.

1. The first stage is relatively short and is characterized by an equilibration in the number of hypocrites of both types. (i.e. $\rho_{Rb} \approx \rho_{Br}$)
2. The second stage is a random walk along an attractor. The walk ends by reaching one of the two consensus states.(i.e. $\rho_{Rb} = \rho_{Br} = 0$ and either $\rho_{Rr=0}$ or $\rho_{Rr} = 1$)

Equalization of the number of hypocrites

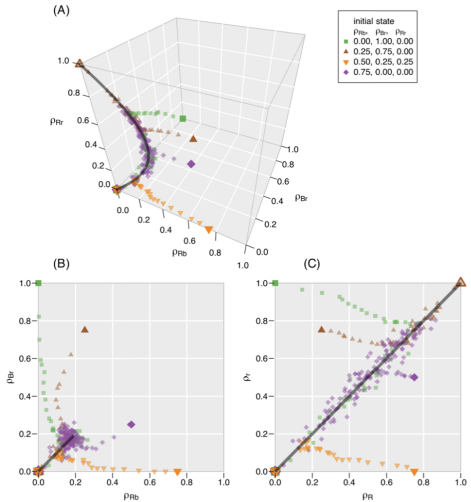

Figure 3: Changes in the composition of the group over time in four realizations of the CVM

Equalization of the number of hypocrites

Equation 6 shows the trajectory equation of ρ_{Rb} and ρ_{Br} with respect to ρ_{Rr}. (Plotted as a black curve in Figure 3).

$$\rho_{Rb} = \rho_{Br} = \frac{\sqrt{(e+i)^2 + 4c(c+e+i)\rho_{Rr}} - (e+i)}{2c} - \rho_{Rr} \tag{6}$$

Denote the difference in the abundances by

$$D = \rho_{Rb} - \rho_{Br} = (\rho_R - \rho_{Rr}) - (\rho_r - \rho_{Rr}) = \rho_R - \rho_r \tag{7}$$

As we mentioned previously, two kinds of hypocrites tend toward equality in the first stage of consensus formation.(i.r. $\bar{D} \to 0$)
Define the absolute value of difference in the abundances as $\bar{D}(t)$. We have:

$$\bar{D}(t) = D_0 \cdot \exp[-(e+i)t] \tag{8}$$

Where D_0 is the difference at time $t = 0$. Let us definie the equalization time T_{equal} as the time t saisfying $\bar{D}(t) = D_0 \cdot \exp(-1)$. Then, we have:

$$T_{\text{equal}} = \frac{1}{e+i} \tag{9}$$

Equalization of the number of hypocrites

The precise ratio of $T_{cons}^{(CVM)}$ to T_{equal} depends on the initial conditions and the parameters c, e, i and N. $T_{cons}^{(CVM)}$ increases linearly with N, but T_{equal} is independent of N.

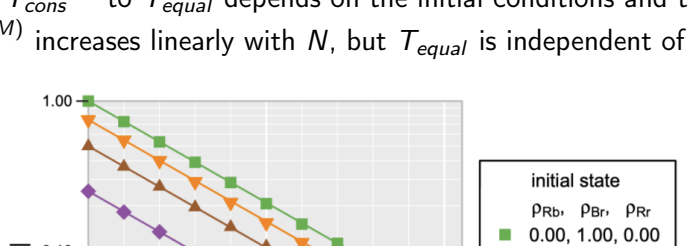

Figure 4: Equalization of the two kinds of hypocrites over time

Equalization of the number of hypocrites

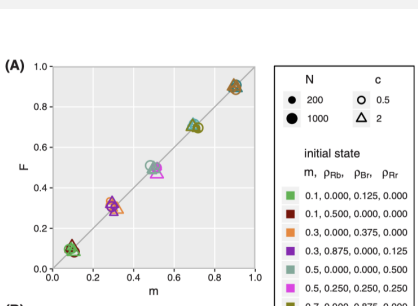

Figure 5: The consensus distribution and consensus time in the CVM

A. Simulations with different initial conditions product the same F if the initial m is the same, despite different abundances of hypocrites.

B. For different N and c, the values of $T_{cons}^{(CVM)}$ fall on different curves. However, for given N and c, $T_{cons}^{(CVM)}$ depends only on m.

How to shorten or prolong the consensus time

- Both $T_{cons}^{(CVM)}$ and $T_{cons}^{(BVM)}$ are increases linearly with N.
- Increasing e and decreasing i can enhance the consensus time.(increaseing e means encouraging **self-expression**, in real life, it's very hard to decrease i.)

Summary

- The interplay between the layers slows down the CVM compared to the BVM, but the number of hypocrites is not directly responsible for the deceleration.
- Both m and $T_{cons}^{(CVM)}$ are determined by ρ_R and ρ_r alone.
- Hypocrisy necessarily emerges in such a system as a path from one opinion to another, but the amount of hypocrisy does not drive the consensus process.

Outlook

- Considet the non-binary opinions.
- Study non-symmetric competition between the opinions.(social media effect will causes bias in one direction)
- **Consider the interplay between internal level**(The external level is a complete graph, for internal level with a very realistic assumption, perple are willing to share their true(internal) opinion with their close friends. Therefore, people can interact within a certain range in the internal level).

References I

[1] Leon Festinger. *A theory of cognitive dissonance*. Vol. 2. Stanford university press, 1957.

[2] Michael T Gastner, Beáta Oborny, and Máté Gulyás. "Consensus time in a voter model with concealed and publicly expressed opinions". In: *Journal of Statistical Mechanics: Theory and Experiment* 2018.6 (2018), p. 063401.

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[4] Richard A Holley and Thomas M Liggett. "Ergodic theorems for weakly interacting infinite systems and the voter model". In: *The annals of probability* (1975), pp. 643–663.

Thanks for your attention

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